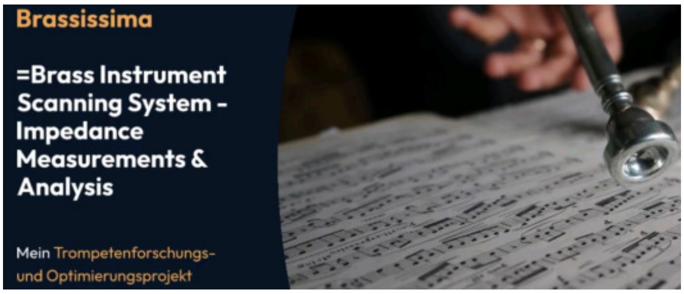
Sideletter #6: Perturbations on a closed-open cylindrical tube giving max. Pitch- and |Z|in Magnitude Change, done with Openwind Simulations.

19.9.2024

This report is a part of fundamental experiments, made with simulation software and also measurements done with a measuring head on simple tubes and complete brass instruments at my private trumpet research project called brassissima. The whole research and documentation can be found at www.preisl.at/brassissima/



The Openwind project:

Openwind
Python library assisting instrument makers

can be found here: https://openwind.inria.fr/

Content:

Description of the experimental setup and applied changes / simulations:

The experiment consists in simulating and measuring a cylindrical tube that is closed on the left end and open ath the right end. The initial size is length = 1000mm and constant inner diameter = 10mm.

The question that should be anwered is how to pertubate such tube, to have a max. frequency change of one of the resonant peaks. Positions given are always the center position of the perturbation from the closed end.
DK is used to describe Positions of Pressure Nodes, DB is used to describe Pressure Antinodes.

Perturbations are of the sleeve type sort, changes are radial symmetric, meaning the circular form stays but with changed diameter (and radius), so inner lateral surface ratio are proportional to diameter ratios.

Mode #3 was choosen for this experiment. ¼ wavelength equals here to 200mm, 1/8 wavelenght to 100mm, so it should be easy to see changes in diagrams. The change in diameter is here limited (wanted) to be +/-10%, giving a ratio of q0=1,1 and alternatively 1/q0= 0,9; remark the the invers proportional change would be smaller: 0,90909.

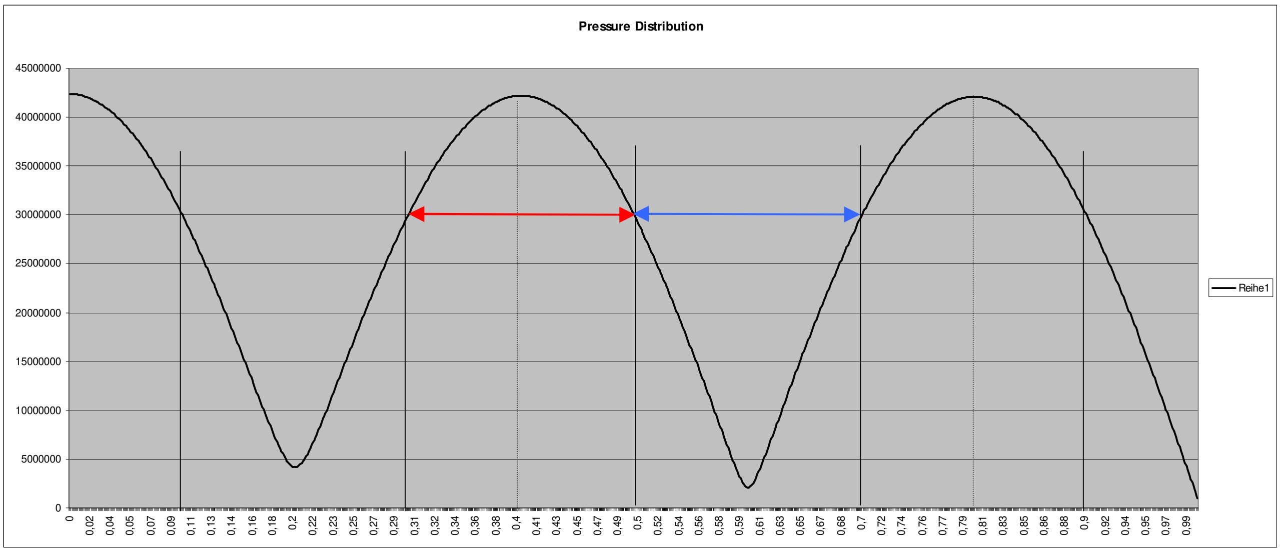
Max. Pitch change (without changing the tube length or temperature):

A perturbation length of = ¼ wavelength, centered at pressure nodes or centered on pressure antinodes should give a difference of resonant frequency

= sin (pl pot) * Xe *100= pot down in %, of difference length of *k* (see sideletter #3, Hal)
= sin (pl pot) * Xc *100= pot up in %, of *k*, (2 pi / wavelenght) = angular wavenumber

PL pot is the ratio of Perturbation length to ¼ wavelength, in this case = 1,0 and sin (1)=1,0
so the Pitch pot = 1* Xc *100 up and Xe*100 dn, if change would be inv. prop.,
Xc (up) being 1/q0² weaker then Xe (dn), Xe here equals = q0²-1 = 0,21 *100 = -21% (pot dn)
Xc here equals = 0,21/q0² = 0,1735 *100 % = +17,35% (Pot up).

k of Mode #3 = 2pi/WL 0,8 = 7,85 rad
Diff k +/- 0,2 ~ 7,65 or 8,05 =~ FF 0,975 or 1,025 = ~ -/+ 43 Cent per full perturbated ¼ WL.

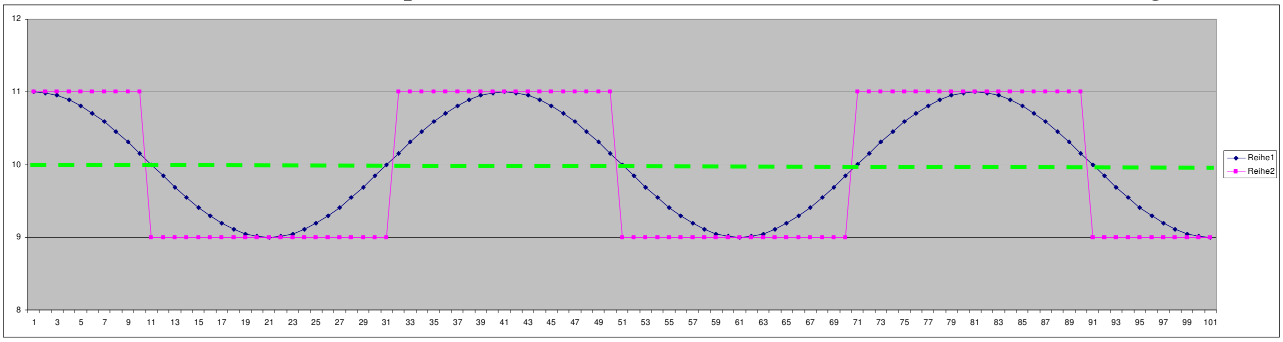


At Mode #3 the tubelenght equals 5/4 waveleghts, Pressure Nodes at 20 and 60% of tube length.
red arrow = ¼ wavelength, centered at XM-IN1 = last DB before 50% tube length,
blue arrow = ¼ wavelength, centered at XM-IN2 = first DK after 50% tube length,
black lines at center positions, where small (and short) perturbations do not change pitch = Pitch-Nodes.

A constriction centered at a Pressure Node (DK) will lower the frequency of the mode,
an enlargement centered at a Pressure Antinode (DB) will also lower the frequency.

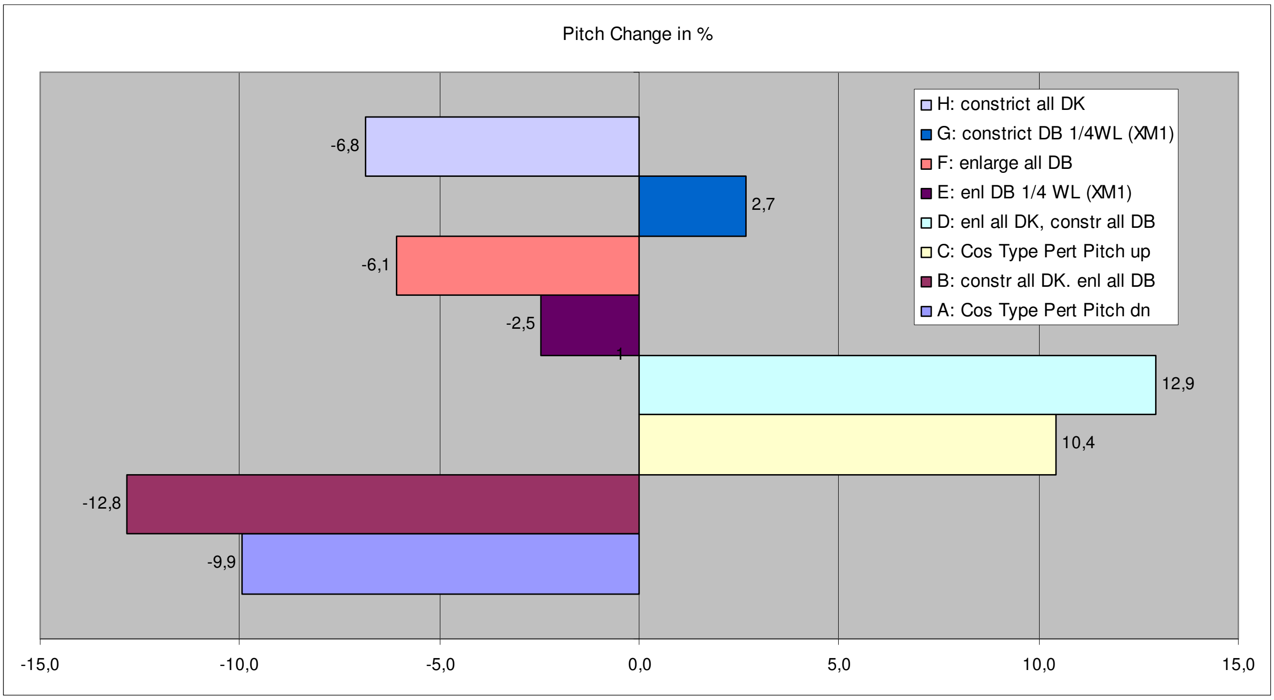
So to find a perturbated bore, where frequency is strongly shifted down, would be a cos function,
where cos = 1 at all Pressure Antinodes (DB), 0 at all Pitch Nodes and -1 at all Pressure Nodes.

The cross-section of the so full perturbated tube would look like this (inner diameter change):



A: Blue: y = diameter in mm of the cosinus perturbated tube, lowering the frequency of mode #3
B: pink: y = diameter in mm of the max. area perturbated tube, lowering the frequency mode #3 max.

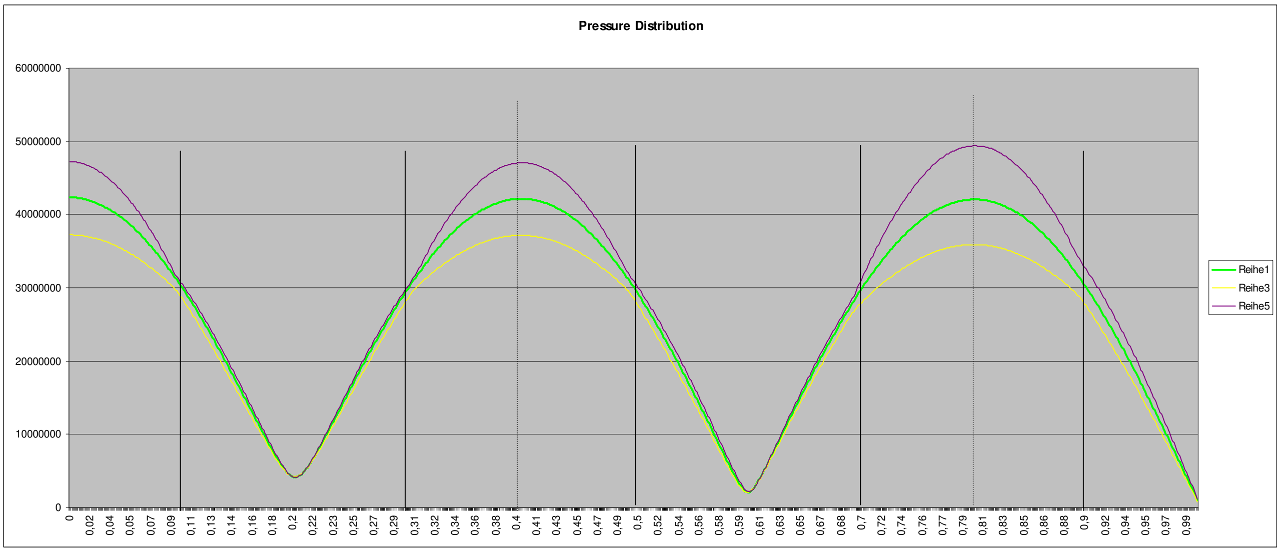
One (wanted) restriction in this experiment is, that the diameter changes should be limited to +/-10%, so the strongest pos. perturbation would be the pink one, (bore-steps), because pitch change correlate with cross section area changes, and the pink is the maximum area change (volume flow) in this case.
For raising the frequency, the sign of cos simply has to be changed to minus.



the cos functions A: / C: give about 10% Pitch Change – this equals to -180 and +172 Cents
the full functions B: / D: give about 13% Pitch Change - this equals to -238 and +210 Cents.
B: “full” Perturbation Pot down is 1,29 times stronger, D: “full” Pert. Pot up is 1,24 times stronger.
E: and G: here is ¼ wavelength (full) perturbated, centered at XM1: giving about -43 and +46 C,
F: here only enlargements (full), are applied, no constrictions for further lowering the frequency
H: here only constrictions (full) are applied, no enlargements for further lowering the frequency
Note: since constrictions here are stronger than invers prop., pitch pot up is also slightly stronger
F+H: together give the full pot down of B: (the partial constr. or enl. pots up are not shown).

2,5% *5 (1/4) wavelengths pot = would be 12,5% 2,7*5=13,5%, so the Pitch Pot of the fully perturbated tube is less (by a small amount). E: and F: -2,5% * 2,5 (1/4) WL = 6,25% in contrast to 6,1% with all pressure antinodes perturbated (enlarged). Single Boresteps would give a pitch change of max. +/- ~ 22 Cents, or (1,3%) this is the half pot of ¼ WL full perturbated (E and G).

Looking at the pressure distribution (changes) – with the full perturbated tube:



Green: unperturbated tube, Dia 10mm, 423,9 Hz Peak Frequency
Yellow: full perturbated tube example B: 369,5 Hz Peak Frequcency = *0,862 /1,160
Violett: full perturbated tube example D: 478,6 Hz Peak Frequency = *1,129 / 0,885

Something strange happens: The frequencies must have diffenent wavelenghts, when the speed of propagation would not be changed. As we see, they share the same wavelength inside the tube (those of the unperturbated tube); being 0,8 meters and ¼ wavelength stay at 0,2m.

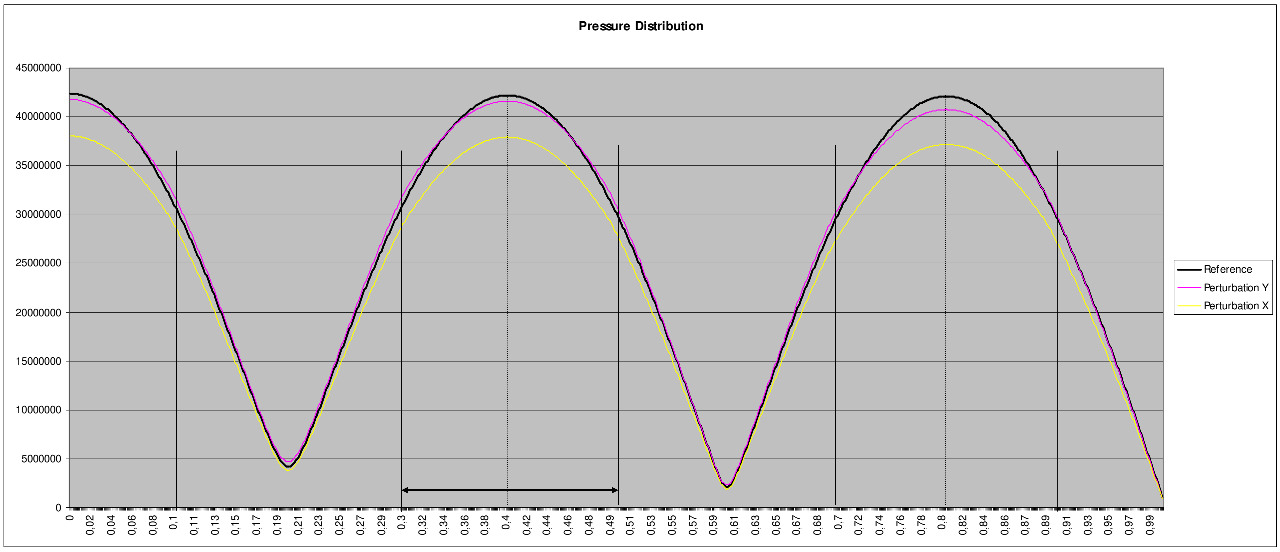
Yellow: a lower frequ. would have longer wavelength in free space; means that here now the wave velocity is higher, Violet: higher frequ. would have a shorter wavelength, but here the wave velocity is lower than the average wave velocity of the unperturbed tube (and in free space).

Assuming an average wave velocity to be equal to speed of sound in free space, and that wavelength is invers prop. to frequency, yellow wave velocity is 1,16 times higher, violet wave velocity is 0,855 times of average speed of sound in air, due to the full perturbation. It should be noted that, with the cos function perturbated tubes, the wavelength found inside the tube is also not changed, so the average wave velocities are also changed, but because of the less pot. the change in speed is also somewhat less.

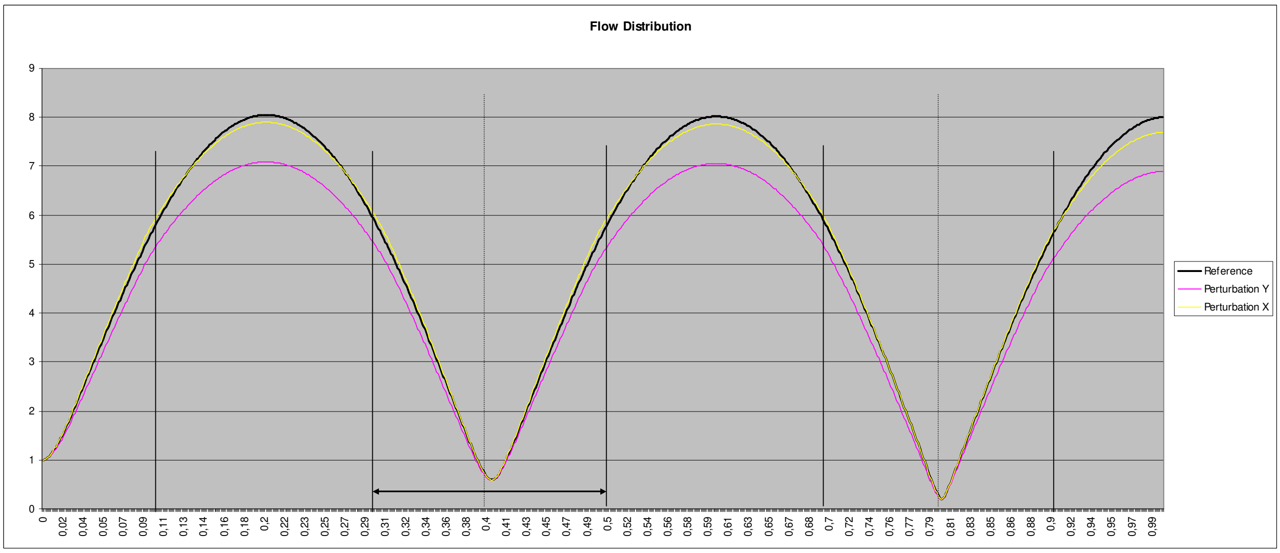
The pressure at all Pressure Antinodes is raised with the raised frequency, and dimished with the lowered frequency, because here also the “entrance area” at the closed end was changed, the found value is prop. to 1/q0 of the entrance diameter ratio, and Zc ratio changes are prop. to 1/q0²:

Reference Input 10mm = Zc = 5,229 MOhm,
A: and B: yellow 11mm = Zc = 4,321 Mohm, q0²=1,21 such a cyl. tube gives lower peak magn.
C: and D: violet 9mm = Zc =6,456 Mohm, q0²=1,234 such a cyl. tube gives higher peak magn.

-> This input magnitude changes in same lowering/raising directions are not the case, if for example only ¼ wavelength is perturbated and the input stays the orgininal reference diameter, here directions of magnitude pot are reversed (and much smaller).



Black: unperturbated tube, Dia 10mm, 423,9 Hz Peak Frequency
(F:) Yellow: only Pressure antinodes full enlarged 398,2 Hz Peak Frequcency= *0,939 /1,065
(H:) Violet: only Pressure nodes full constricted 394,7 Hz Peak Frequency = *0,931 /1,073
Wave velocity is higher than in the unperturbated tube, about 7%
H: Since the entrance area at the closed end is here not changed, and positions of the highest pitch pot are (almost) positions of Input Magnitude Nodes, the magnitude change here is very small.

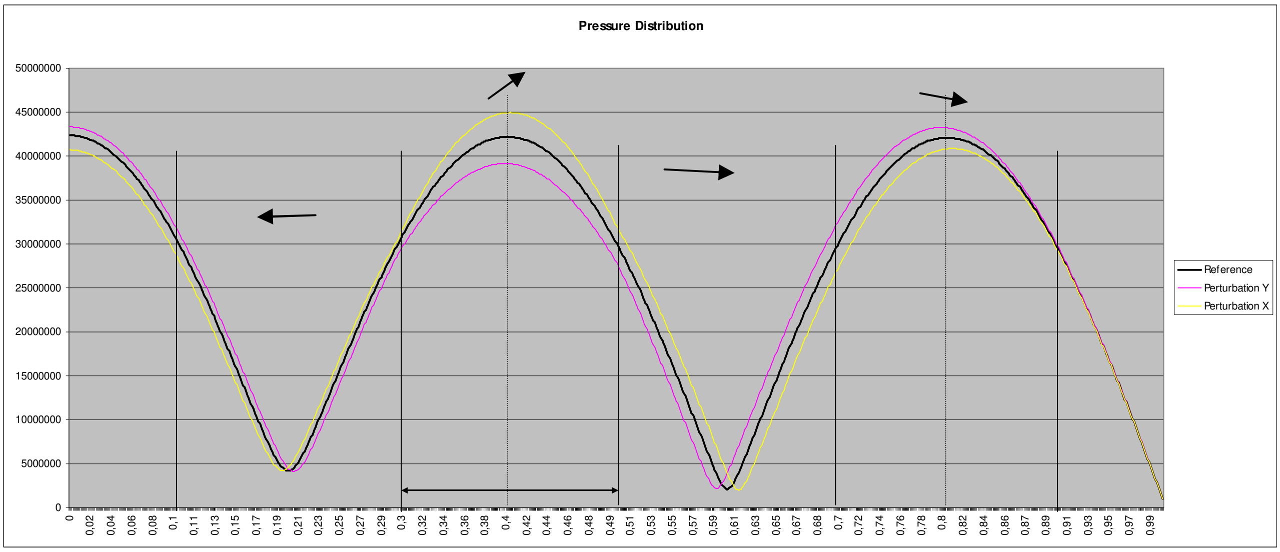


H: since pressure nodes are antinodes for flow, here the flow is lowered at the constricted parts.

The question arises, if that is the full pitch pot, or will there be a situation, where wave velocity is changed less, this would mean, that at least one pressure antinode should not be perturbated, so that its position can change:

The short answer is: Full perturbation gives the most pitch change, but is somewhat smaller than the sum of single ¼ WL Perturbation lengths. Single Boresteps give a pitch Pot (but on positions where those are pitch nodes with local perturbations!!), which is ½ that of ¼ WL perturbated. If the closed end area is unperturbated to some degree, the pitch pot is smaller, there are different answers how impedance input magnitudes are effected and positions of pressure nodes and antinodes are shifted.

Situation, when Perturbations are <=1/4 WL, centered at Pressure Antinode (or Nodes, reverse)

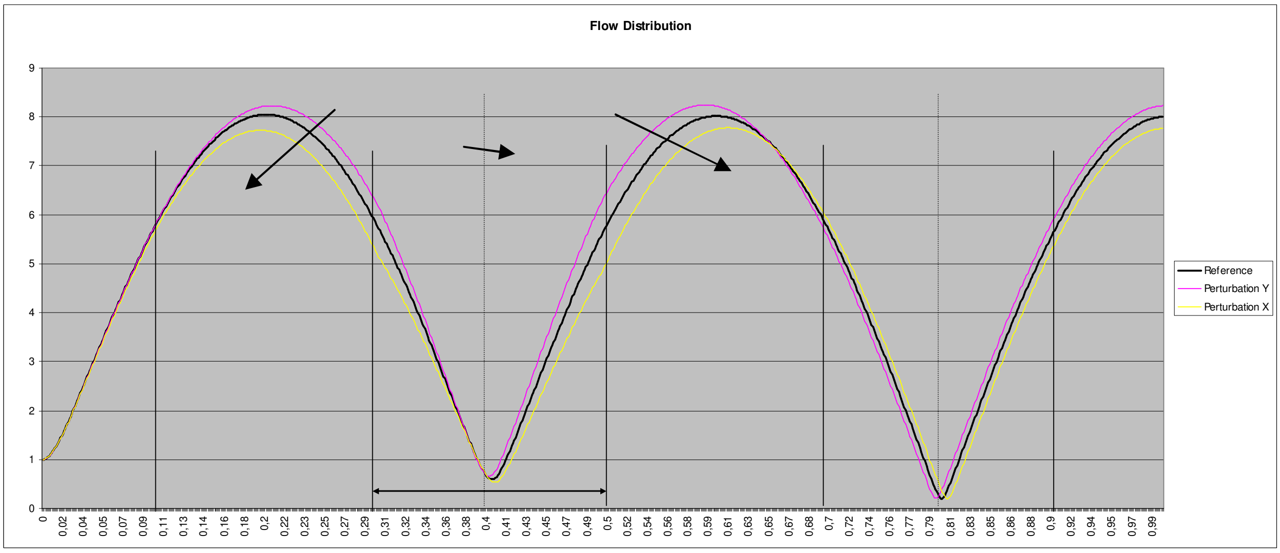


yellow=G: ¼ WL centered at XM-IN1 = Pos. 40% tubelenght, constricted = Pitch +2,7% = *1,027
Pressure Nodes before and after the pert have a longer distance = 0,613-0,195 = 0,418-0,4=0,018m

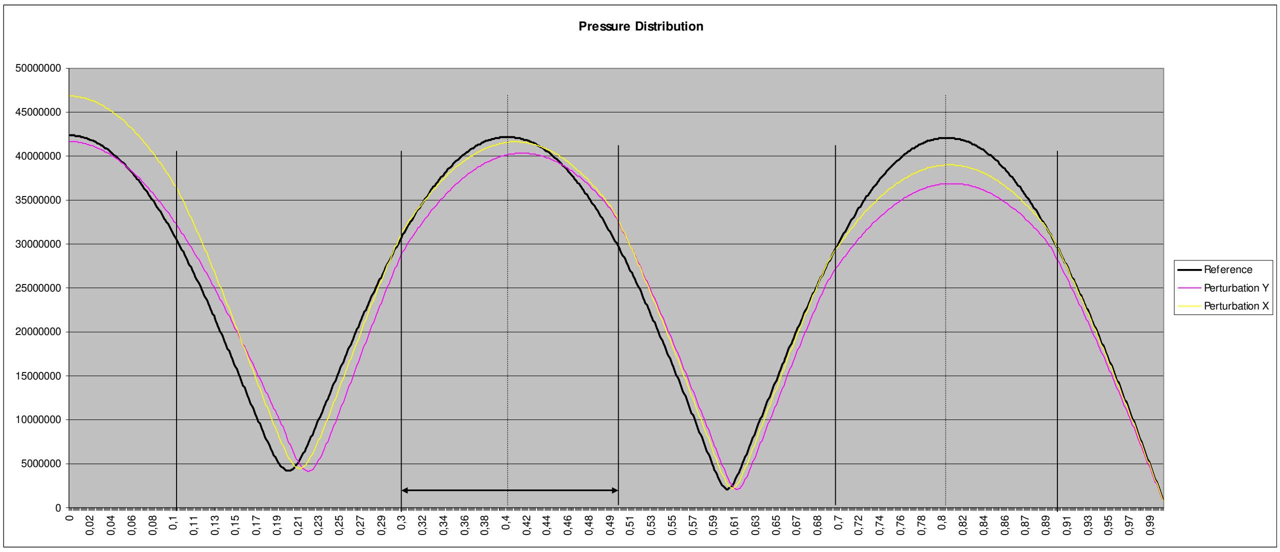
0,2 -,0195 = 0,005m shorter than 1 1/8 WL, 2x 0,009m longer than 1/8WL, 2x0,0065m shorter
1x -5mm, + 2 x 9mm=+18mm - 2 x 6,5mm =-13mm

If the wave velocity would not be changed, the wavelength would be 1/1,027 = 0,9737 times of 0,8m = 0,779m = 0,021m = 21mm shorter. ¼ WL would be 0,19475m.. (4,2mm shorter)
so the distance from closed end ~ equals nearly the shorter ¼ WL.
The distance from open end to last pressure node equals nearly 2 x shorter ¼ WL. There is a difference of ~3mm, this is within the accuracy of reading the found nodes and partially changed wave velocity around the perturbated region.

pink: =E: ¼ WL centered at XM-IN1 enlarged = Pitch -2,5%
I did not calculate this, but it should be nearly the same, global wavelength gets longer, enlarged section "shrinks".



Flow distribution is changed in the same direction as is pressure distribution.

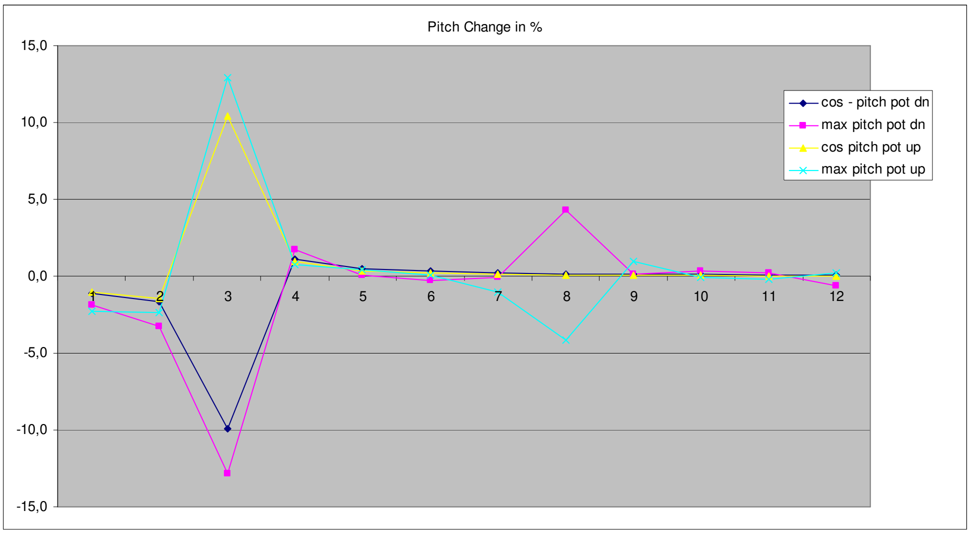


Yellow: First 10% of tubelenght at closed end are not perturbated, then full perturbated, 374,9 Hz
Violet: First 20% of tubelenght not perturbated, perturbation starts at pressure node, 379,2 Hz
in both cases, the frequencies are lowered = longer wavelength, but pressure nodes are smaller spaced. Only at the closed side, the first pressure node is therefore placed away from the closed end.
So in this case, also the wave velocity must be larger – it seems to be largest around the pressure antinode at XM-IN1, here at ~30% tube length.

Yellow: First 10% of tubelenght at closed end are not perturbated, then full perturbated, 374,9 Hz
Violet: First 20% of tubelenght not perturbated, perturbation starts at pressure node, 379,2 Hz

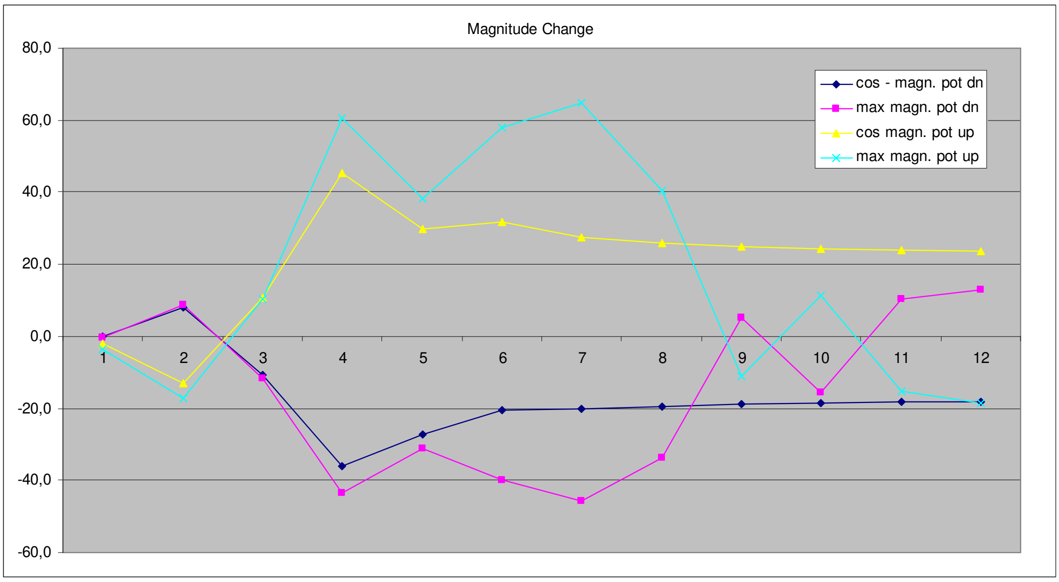
For showing the effect between full and “cos” type perturbation, constrictions are used, with are somewhat stronger, than the enlargements, the inv. prop. constr. Diamenter would be 9,0909mm.

However, this geometry and experiment is here specially formed and prepared for mode #3, here are the results on the other modes how weak the pitch pot for other modes is:



The shown data is Example A,B,C and D where the whole tube is perturbated.

Mode #3: has a quarter wavelength of 1/5 tube length = 20,0%,
Mode #8: has a quarter wavelength of 1/15 tube length = 6,66% = 3,0 times shorter,
and is also strongly influenced, since the "pattern" does match to some extend: However, the Freq. change is only about 50% that of mode #3, and then only with the "full" perturbations.

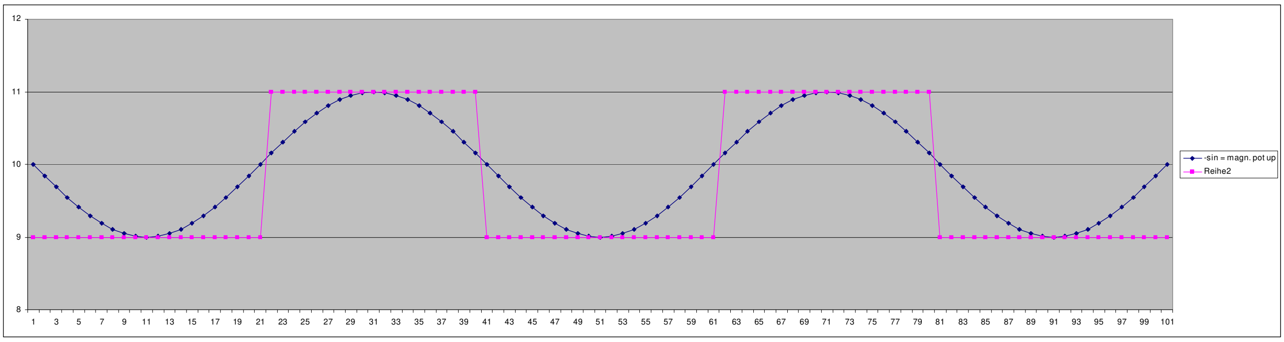


Magnitude changes do not match the pattern, however interesting is, that all Modes above Mode #3 are changed in the same direction, being more than twice as strong changed.

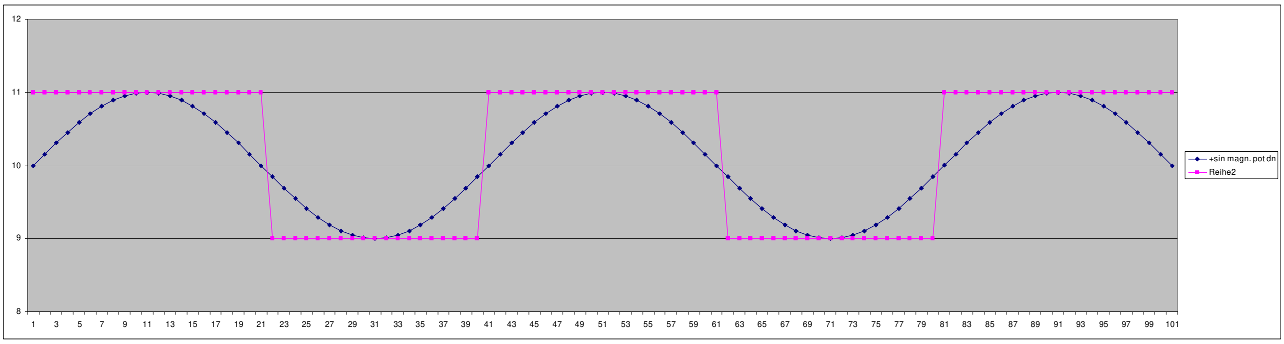
Max. Input Magnitude |Z| change:

As additional task, the max. possible Change in Input Magnitude |Z| of Mode #3 was searched.

A strong change is found, when now the –sin function is used for raising the Peak Magnitude. This would place zero crossings on Pressure Nodes and on Pressure Antinodes. As with pitch changes, the max. Pot is found, when the sin curve is replaced by steps giving the most cross section change between zero crossings. Vice versa, Magnitude lowering pot is found by replacing –sin by +sin.



Such change in Diameter (pink), (closed end is at the left side) would give max. Magnitude Peak shift upwards for Mode #3. At pressure nodes and pressure antinodes the diameter is not changed (or in the full perturbation case it is the zero crossing of a borestep).

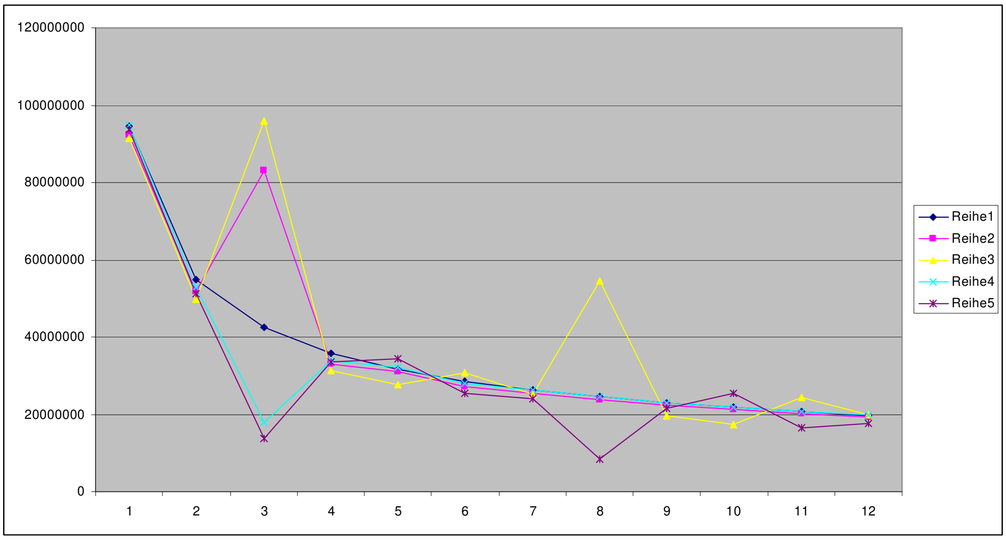


This would be the corresponding shape for max. lowering the peak magnitude of Mode #3.

Magnitudes behave not similar to pitch. On the open side of the tube, the crossings found do not match Pressure Nodes and Pressure Antinodes anymore. However, since the pot is getting very small in direction to the open end, the deviations are also small.

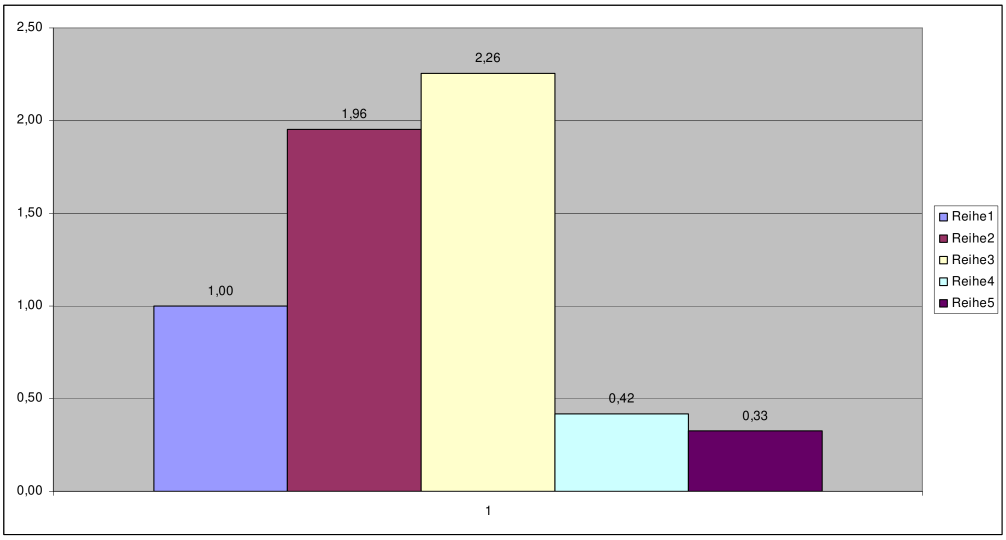
We can not name the pot of ¼ Wavelenght anymore, it could be replaced by 1/8, 3/8, 5/8 and so on Wavelenght pot, seen from the open end to the center of the perturbations. There is an “endpot”, end the pot is growing toward the closed end.

Since (my) physical impedance measurements depart here from simulation models to a large degree, the following simulation results should be seen to be “hypothetical”.



1. blue = reference,
2. pink: -sin curve perturbation over the whole tubelengt, 3. yellow: max. full perturbation
4. turquoise: +sin curve pert., 5 violet: max full perturbation, Modes #1-#12

As with pitch, the other mode magnitudes are not strongly changed, also Pitch (Mode #3) is almost not changed, but as with pitch, Mode #8 shares some pattern – (when fully perturbated), the wavelength is here 3,0 times shorter



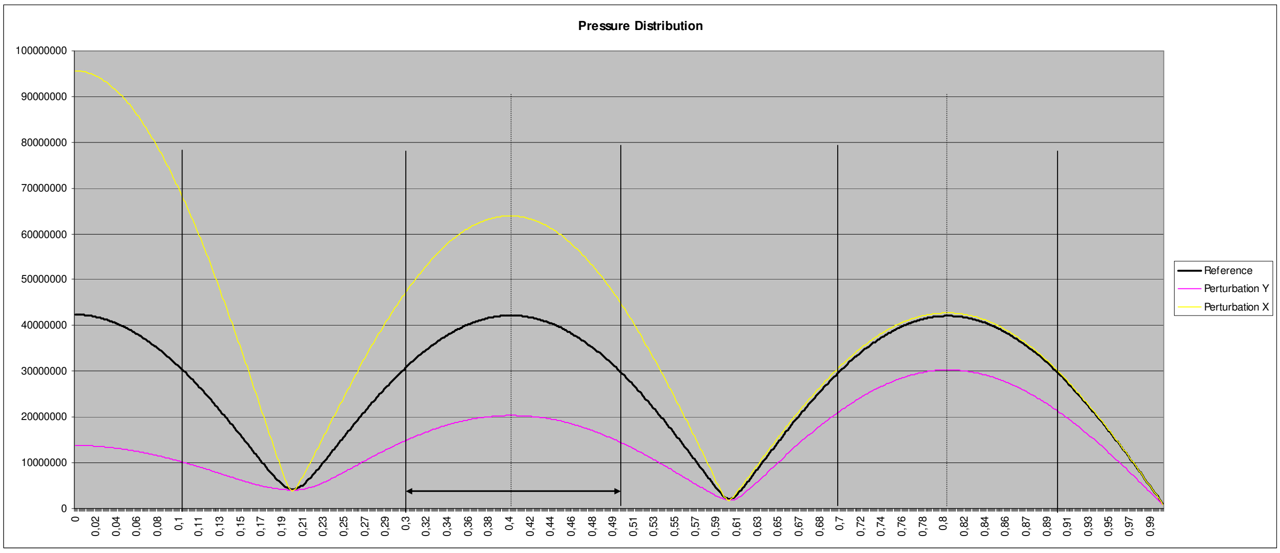
Peak Magnitude change als ratio to reference, Mode #3
1 reference, 2 –sin pot up, 3 max pot up,
4 +sin pot dn, 5 max. pot dn.
Sinus perturbation type: Magnitude lowering is 1,21 times stronger than raising, ~q0^2
Full perturbation type: Magnitude lowering is 1,33 time stronger than raising ~ q0^3

In the open wind simulation model, the full perturbation type with max. +/- 10% diameter changes is able to push the magnitude of peak #3 more than 2times, and to reduce it 3 times – which would equal a 2 times diameter smaller or a 3 times diameter larger unperturbated tube!

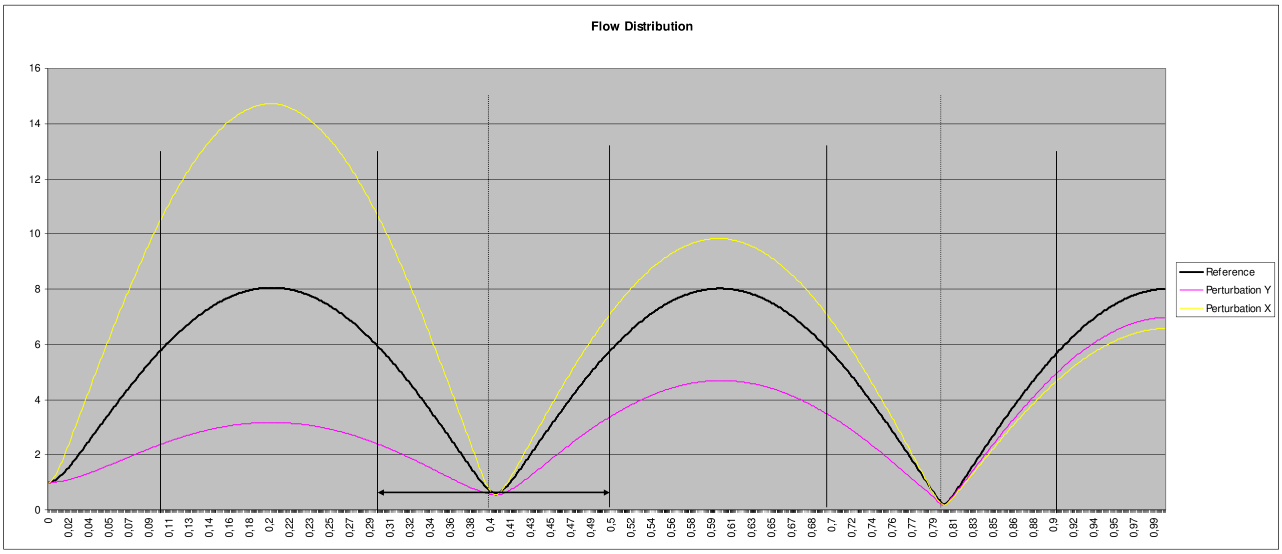
As stated, inversely proportional constrictions should be smaller to be compareable with enlargements, or vice versa. So in this setting constrictions have a stronger pot. than enlargements. Instead of 90%, the constricted cross sections should be only 90,909%, or instead of +110% enlargement it should be + 111%.

In the magnitude lowering case, you maybe see, that (too large constrictions) are always on raising pressure antinode peak flanks (left), and this means additional stronger INVERSE Magnitude Pot, than (too small) enlargements on the falling pressure antinode peak flanks (right). In the magnitude raising case it is vice versa. So the difference in changes would be somewhat less than shown here, as the cross section difference was kept at +/- 10%, to simply compare the sin function and its potential against the most possible solution (boresteps perturbations with sharp edges).

It could be modelled what the result would be when only the resulting inverse or non inverse regions are perturbated, I did this not at this moment, because there are some open questions how those differences are originated at all, simulated and measured.



Yellow: 3 – max. Input magn. up bore profile, pink: 5- max. Input Magn. dn bore profile
Open Question: Why is here the change of pressure distribution at the last pressure antinode (80%) so different?



Yellow: 3 – max magn.up bore profile, pink: 5 - max Input Magn. dn bore profile

The pressure distribution along the tube with its decreasing amplitudes looks very like the distribution found in a cone, but in the cone/frustum additional the pressure nodes are shifted left which gives a strong change to (higher) frequency.

Notice: The pressure and flow amplitude envelopes are not linear, its some type of exponential or power function.

When there would be only steps at pressure nodes (here 2, at 20 and 60% tubelenght) AND all are in the same direction; the pressure distribution would stay linear but on a higher or smaller value troughout the tube.

There are some striking questions open, if the simulated results are valid:

Global Pitch:

A single local perturbation (Enlargement) of ¼ wavelength center placed on a pressure antinode, mode #3 at tube position = 0,4 (XM1) fore example would have ~2 times more – but inverse - pitch pot change, (about 44 cent) compared to

a single (positive) borestep placed on a descanding pressure flank;
(at position 0,5 times tube length), but also at 0,1 and 0,9 times tube length in the case of mode #3.
We find, the distance between these positions with same pitch pot is always 1 full wavelenght.

With boresteps there is a pitch pot relation to 1/odd # = wavelength.
A single positive borestep at 50% tube lengths equals 1/8 WL from the open end for mode #1,
the pitch pot with a positive step to 10% more diameter gives a change in Freq. of ~ +110 Cents.
Mode # 2, (3/8 WL) at 50% tube length gives here a inverse pot of ~1/3 of that, Mode #3 gives a pot of 1/5 of that.
So the max. possible pitch pot of mode #3 equals ~ +/-22 Cent with a singe borestep here.

With local perturbations it is found, that pitch lowering pot is always $q0^2$ times stronger than raising pitch. This is not the case with boresteps, the differences in logarithmic cents are almost same, so here it shows an invers prop. behaviour up/dn = not stronger down.

If the whole tube would be enlarged, pitch of mode #3 would raise about only 2 Cents, and would be -2,3 Cent lower with a constricted bore, inversely prop. to the enlargement. So its already non invers pitch behaviour. But there is a change in losses calc., because the area where losses to the wall are assumed being compareable stronger with the smaller tube. We find, a pitch pot in this case is almost cancelled – or describe it as the effects of losses and endcorrection at the open end.

With single local perturbations and their possible pitch pot there is a relation to one perturbated section against all the non perturbated sections, giving the same pitch pot changes (in cent) = logarithmic … for all modes. – so this is inverse. It is also inverse, that the distance of resulting pressure nodes around the perturbation gets smaller, giving the inverse result of longer (global overall) distances, meaning lower resulting global frequency and vice versa.

In a simple cylindric tube it is assumed, that the “average” wave velocity is ~ equal to the “average” velocity = speed of sound in free space.

So it comes, that in a cylindrical tube whith local perturbations, the wave velocity is not equal to the wave velocity in free air. Therefore the distance of found pressure nodes is compared. If two nodes are closer, the velocity between them would be higher; or vice versa. Such local higher speed (around the perturbation) gives room for a lower speed before and after, and as result a lower frequency of the mode where the possible magnitude is achieved.

Input Peak Magnitudes Modulus/Betrag |Z| at the acoustically closed end:

The reflection factor tells how much pressure pot is lost. In a closed-open cyl. tube there are 2 full roundtrips necessary to complete the jorney being in same phase, but since we have pressure input in form of sinus shape, there are ½ WL (and) ½ Periods where the backward travelling wave gives superposed standing waves with the forward travelling (and) starting waves. The reflection factor (difference of it = loss) ist spread over the tube, those losses appear most lineary, the pressure amplitudes are dimished at every pressure antinode with the same amount, being RF*2. However, they are not linear, but we find the average as pressure antinode amplitudes at the closed end, (and at all pressure antinodes, if there are no perturbations along the tube).

Sideletter #2 has been enhanced and the RF and SWR ratio behaviour better explained in detail.
Part 2E 2 Modematching describes the general behaviour of positive boresteps:
Placed at pressure nodes, Z magnitude is strongly raised, but at pressure antinodes Zin is only minute lowered.

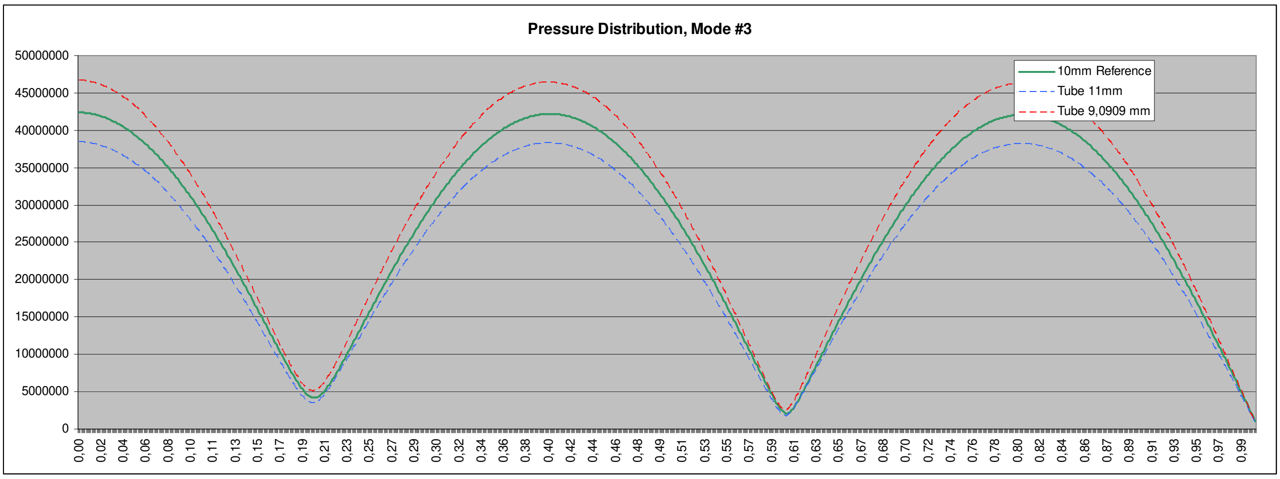
Whe should say that the reflection factor now is not anymore the average, but is different before and after the step; as also Zc is different; in * $q0^2$ (area) enlarged tubes Zc it is $1/q0^2$ smaller, so its time to look, what happens with SWR and RF in different Tube Diameters, and in Tubes that have a (single) borestep (or more steps) at pressure nodes:

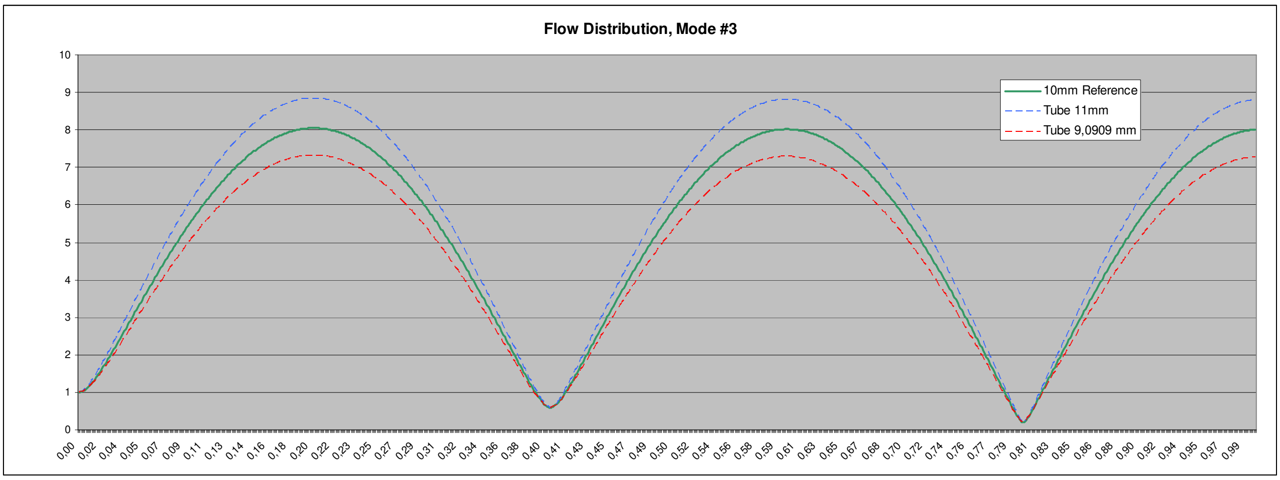
We look first on the openwind simulation results of pressure and flow distribution in the case of <u>full</u> boresize change:

Unperturbed closed-open cylindrical tubes with length = 1,0m openwind results, Mode #3 (5/4 wavelenghts):
Since openwind normalizes the pressure p to Flow U = 1,0 max. Press.Ampl. = |Z|in Magnitudes = p / U

max. Pressure: Dia 11mm 38,42 = /q0 =1,1 Zc=4,322 Mohm = /q0² = 1,21 SWR=8,889 RF=0,797 ZMin: 0,486
Dia 10mm 42,39 = Ref.=1,0 Zc=5,2295 Mohm = Ref. = 1,0 SWR=8,105 RF=0,780 ZMin: 0,645
Dia 9,0909 46,67 = * q0=1,1 Zc=6,327 Mohm = *q0²= 1,21 SWR=7,376 RF=0,761 ZMin: 0,857

Found q=SWR is 1,096 times (stronger) in the enlarged tube, 0,9100 times (weaker) in the constricted tube, **= ~ q0**
so it is <u>inverse</u> prop. changed compared to magnitude peak changes against the reference tube diameter change.
RF Factor is 1,022 times stronger or 1,7% less losses; 0,975 times (weaker) or 1,9% more losses. (~ 1/5 of SWR)





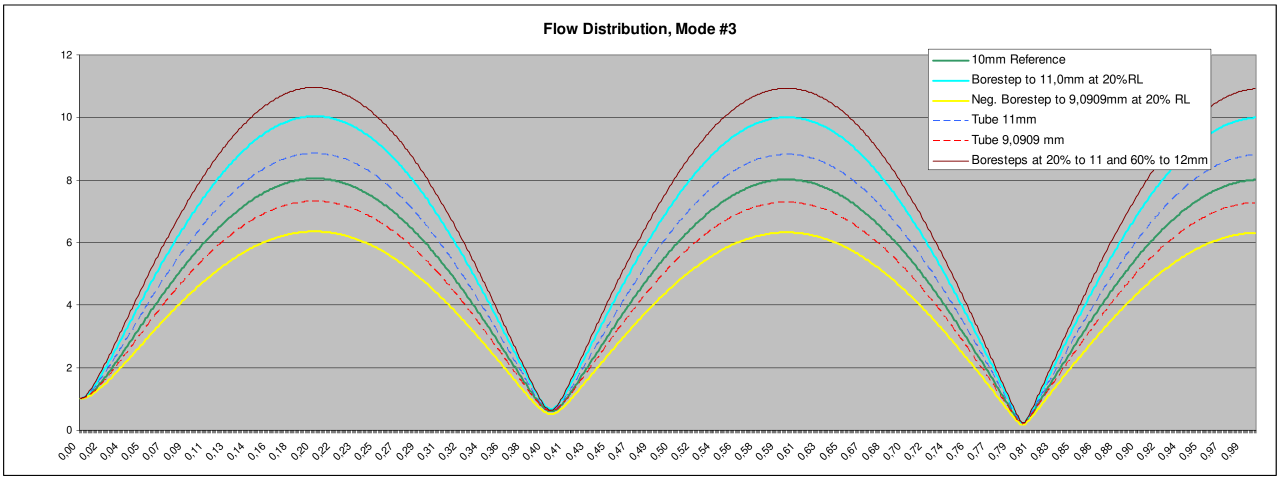
Unperturbated tubes max Flow: Dia 11mm 8,835 = *q0 =1,1 we find more flow, less pressure
Dia 10mm 8,05 = Ref, =1,0
Dia 9,0909 7,315 = /q0 =1,1 we find less flow, more pressure
So, the (normalized) max. Flow Amplitude is a little bit smaller then the SWR-RATIO / q-Factor and
it stays normalized 1,0 at the closed end.

So it can be summarized that: A full boresize change in tube diameter *q0 gives a changed Zc at the closed end, and:
Z Peak Magn. /q0, Zc /q0², Z Magn Min /q0³, but SWR gets = * q0 and the Reflection Factor also becomes higher
with larger tube bore diameter.

We look now on openwind simulation results of pressure and flow distribution in the case of boresteps:

Zc at the closed end now does not change and stays at p*c/S = Zc=5,2295 Mohm.
So, all other values must be changed: Zmax, Zmin, SWR and RF.
As found by systematic testing, the Zin Peak Magnitudes are max. changed, if borejumps are placed at pressure nodes.

I start with flow, as this is a very interesting result:

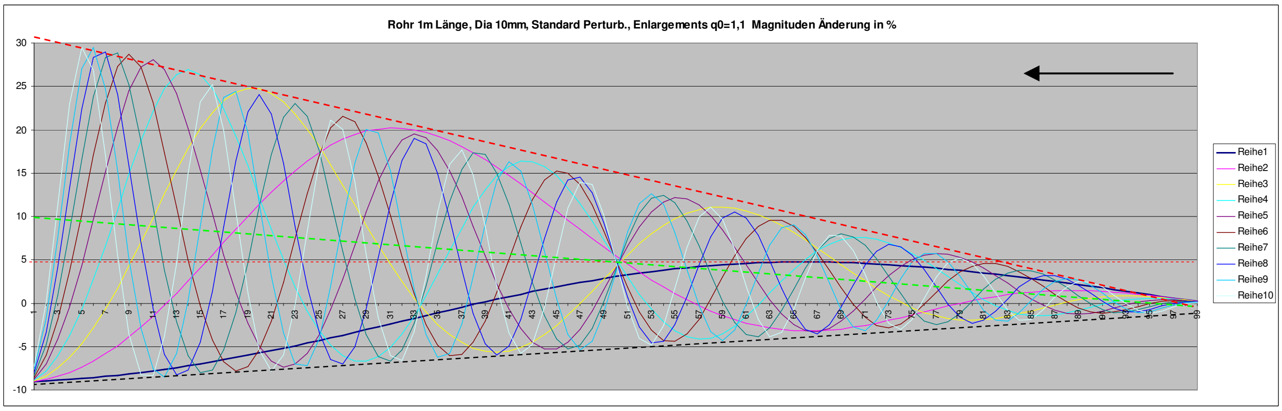


Possible Flow (Envelope) is everywhere the same along the tube, and much higher with the positive borestep.
But if the step ist not at a pressure node, the flow distribution is not anymore the same over the tube ..

Full boresize change: flow*q0, borestep after ¼ of 5/4 WL: = 10,04/8,05 = * 1,25 or= 5 : 4 equals 1/0,8: 1.
0,8 beeing the tube length ratio with larger diameter. gives 1,25 … times more possible flow.
with Constriction after ¼ of 5/4 WL: 6,36 / 8,05 ~ = 0,8 und 1/0,8=1,25 … times less possible flow.
– but q0 is missing here, so that's not the complete answer. Why + 25% or -20% Changes at 20% Tubelenght?

Pressure / Zin Magn.: In a tube 5 times shorter = 1/5 = 0,2m we have 5 times more magn. pot. and 5 times more flow.
In a tube, 3/5 length = 0,6m = 1,666 times shorter we have 1,666 more magn. pot; and 1,66 times more flow.
This simple relation has also been checked with some openwind experiments.

If at 0,2 RL = 1. Pressure Antinode, (Mode #3), the diameter jump is: (---> 10mm unperturbated RF=0,780)
10 to 11 q0=1,1 it gives ~ 1,25 * Magn.+Flow; more then q0²=1,21; SWR=10,13 RF=0,820
10 to 12 q0=1,2 it gives ~ 1,5 * Magn. +Flow; more then q0²=1,44 SWR=12,15 RF=0,847
10 to 14,14 q0=1,4142 it gives 2,0 * Magn. +Flow; = ->> **q0²=2,0** **SWR=16,21** **RF=0,884**
10 to 20 q0=2,0 it gives 3,33* Magn. +Flow; less then q0²=4,0 SWR=26,09 RF=0,928
10 to 40 q0=4,0 it gives ~ 4,66 * Magn. +Flow; much less the q0²=16 SWR=37,8 RF=0,948
so, that it is already the open end, it is 5*Magn. +Flow. SWR=40,5 RF=0,950
this is inverse Peak Magn. Pot. (up), since Magn. Pot. is now * >q0² or * <q0², instead of /q0!



Looking at the summary of positive borestep positions along the tube, we find some interesting aspects:
The black arrow indicates progression of enlargement – starting at the open end
1. non inverse Magn. Pot down is max with full perturbation =1/q0 at Pos.0 =-9,9%, on all pressure antinodes (dn).
2. at 50% Tubelenght, all modes (except #1) are q0 / 0,5 Pot. up = +5% = inverse., so magn. nodes are far away.
3. dotted green line shows mirrored pot. up, = +10% at Pos.0. Further we can notice, that this is the arithm. mean of the found non inverse pot down and the found inverse pot up, and from this (average) mirror, all inverse magnitude pot is not stronger, thann non inverse. So it comes, that finally the inverse pot is 3 times stronger.
There are small deviations to this rule, but they are small. So from the green mirror line, dy+ is the same as dy-.
4. Comparable inv. pot up is found at step pos. at 2/3 tubelenght (1/3 from the open end); so max. inv. pot up is 3 times.

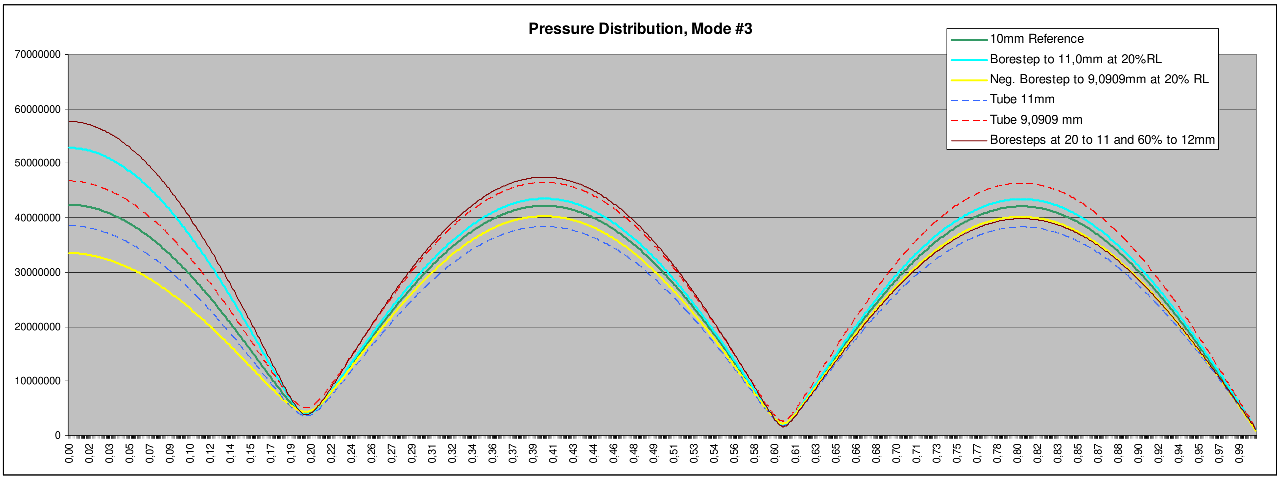
In the case of Mode #3; at Pos 0,2 the mirrored pot up is (1-0,2)* q0 pot = 8%, the found inv. pot is 3 * 8% = 24%.
dy mirrored pot up to pot dn = 2*pot dn = -16%, pot up is dy pot dn + 2 * pot dn = 8+16 %. = TL 24%.
with borestep at Pos 0,6 the mirrored pot up is 0,4* q0 pot = 4%, and the found inv. pot is 3 * 3,66% = 11% (1% less).

Mode 2 at Pos 0,33 has mirrored pot up=0,66*q0 = 6,66% and the found inv. pot up is 3 * 6,66% = 20 %.
As we see from the linear red dotted line, this is nearly exact true for all modes.

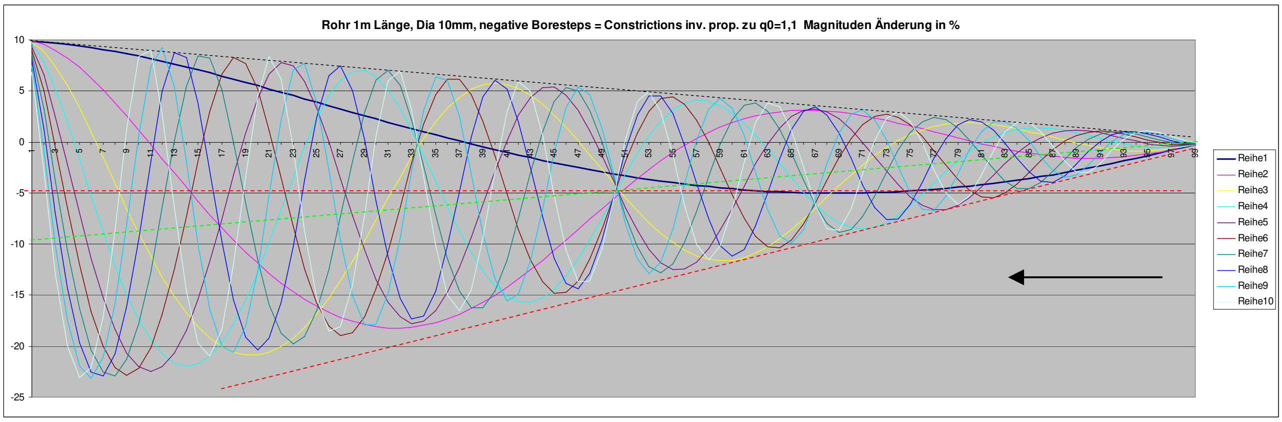
Larger q0:
In the case of q0=1,2 positive step, Mode #3: 0,8*20%= 16%; +16%green =dy=32%dn; 16+32=48% up (~50% found)
in the case of q0=1,4142………… 0,8*41%= 32,8%+32,8 green dy=65,6: 32,8+65,6=98,4% (~100% found)

So we could write, the magn. pot. envelopes of positive boresteps are (in % Input Magnitude change):

Non inverse Peak Magnitude envelope down change = tubelenght 1 – step.position * [((1/q0)-1)*100],
a mirrored pot up gives an arithmetic mean pot (up) = 0-pot dn, so dy from mirror is 2*pot dn, and 2*pot up,
whitch equals (up to q0² <=2), that inverse magnitude envelope pot up is 3 times magnitude pot down.
At pressure antinodes, the pot is non inverse and dn, at pressure antinodes the pot is inverse up, 3 times stronger.



Pressure and Zin Changes are 3 times stronger (inverse up) when boresteps are placed on pressure nodes, compared to changes on pressure antinodes (dn). A linear deviation error to the openwind simulation result is smaller then 2%.
Mode #1 has no Pressure antinode inside the tube, here is an inverse overpot up =pot dn/2, at Position 66% tube length.



Looking at the Openwind summary of negative borestep positions along the tube:
The black arrow indicates progression of constriction – starting at the open end.
Constrictions behave different; Mode #3 max. inverse pot down is ~1/pot of an enlarged tube at the 1st. press. antinode, but at the second antinode it is (not weaker), so the behaviour of the invers pot down is not linear. So with a smaller part constricted inv pot. is here stronger, getting weaker when a very large part ist constricted (from the open end).

Note: The ART Simulation results deviate from the OW Simulations and ART gives more "linear" inv. pot dn.

Summary of this sideletter:

This is a special single part of many experiments and simulations and should be taken as such a very special task. The foregoing fundamental experiments were done, even knowing that they are very hypothetical to a great degree. They are (at first sight) not very usefull for musical instruments but should show where the found limits / maximal changes are found and under what conditions such changes produce a very small effect on pitch / or magnitude |Z|in.

It should simply be answered, how and why “perturbations” of the bore generally work, and that single boresteps are the entrygate to many boresteps, as they can be extended to boreprofiles of “horns”, where they are ongoing along the tube and how they contribute to found problems related as mode-matching and impedance mismatch – being the source of reflections and so the build up of standing waves, that are needed in musical wind instruments, but what are generally not wanted in impedance coupling/matching systems and so within eletromagnetic transmission lines at all.

Full perturbated bores – (optimized for a single resonant wavelength) change the wave velocity along the whole tube.

It should be clear, that full boresize change, boresteps, centered perturbations and local perturbations behave very different on the change of “global” resonant frequency and input impedance – and as a consequence also on the radiated sound spectrum and power.

Pitch Changes are understood to a very good degree, computed input Magnitude values themselve and changes are eventually strongly overestimated in (any) simulations compared to (my) physical measurements, this is part of future investigations.